

Continuity Review

1. $f(x) = 2x^2 - 3$

a. Find the average rate of change of $f(x)$ on $[1,5]$.

$f(1) = 2(1^2) - 3 = -1$

$f(5) = 2(5^2) - 3 = 47$

$(1, -1)$
 $(5, 47)$

$\frac{47 - (-1)}{5 - 1} = \frac{48}{4}$

ave. vel = 12

b. Find the instantaneous rate of change of $f(x)$ at $x = 3$.

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\frac{2(x+h)^2 - 3 - (2x^2 - 3)}{h}$$

$$\frac{2x^2 + 4xh + 2h^2 - 3 - 2x^2 + 3}{h}$$

$\lim_{h \rightarrow 0} 4x + 2h$

$f'(x) = 4x$ $4(3) = 12$

c. Find $f'(2)$.

$f'(2) = 4(2) = 8 \leftarrow m$

d. Find the equation of the tangent at $x = 2$.

$f(2) = 2(2)^2 - 3 = 5$
 \downarrow
 y-value

$y - 5 = 8(x - 2)$

$y - y_1 = m(x - x_1)$

2. Find a value for $f(5)$ that would make $f(x) = \frac{x^2 - 25}{x - 5}$ continuous at $x = 5$.

$\lim_{x \rightarrow 5} \frac{(x+5)(x-5)}{x-5} = x+5$

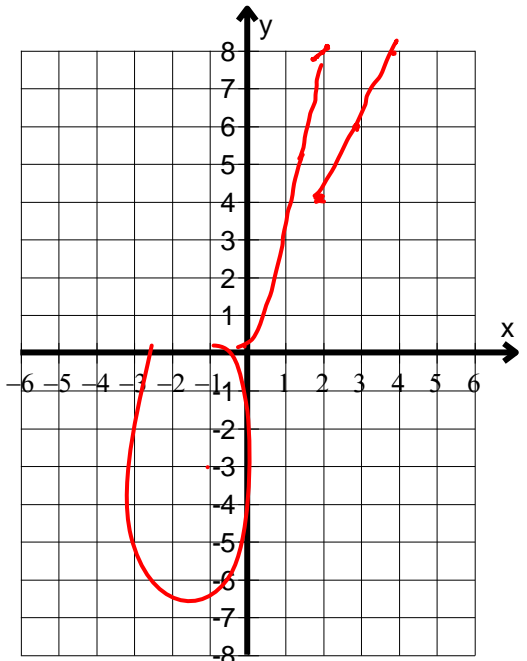
$5+5=10$

3. $f(x) = \begin{cases} 2x-1 & x < 0 \\ 2x^2 & 0 \leq x < 2 \\ 2x & x \geq 2 \end{cases}$

a. Graph the function

b. Determine where it is discontinuous. *at 0 and 2*

c. Determine the intervals of continuity



4. Using the Intermediate Value Theorem – Prove there is a root on $f(x) = x^3 - 3x + 1$ on the interval $[0,1]$

$$f(0) = 0^3 - 3(0) + 1 = 1$$

$$f(1) = 1^3 - 3(1) + 1 = -1$$

By IVT, the signs change on the interval $[0,1]$ proving the graph crosses the x-axis.

5. $F(n)$ is a function which represents the cost in dollars for producing n number of copies of an advertisement

n	10	20	30	40
F	1000	1200	1800	3000

- a. Find the average rate of change on the following interval $[10,20]$

$$\text{Ave} = \frac{1200 - 1000}{20 - 10} = \frac{200}{10} = 20 \text{ copies per dollar}$$

- b. Find the average rate of change on the following interval $[20,30]$

$$\text{Ave} = \frac{1800 - 1200}{30 - 20} = \frac{600}{10} = 60 \text{ copies per dollar}$$

- c. Find the instantaneous rate of change at $n = 20$

$$\frac{20 + 60}{2} = 40 \text{ copies per dollar}$$

- d. What does $F'(n)$ mean?

Change the rate of cost as copies increase

- e. What are the units of $F'(n)$?

copies/dollar